GENNADY A. YANCHENKO (*National University of Science and Technology NUST MISiS*)

THE RELATION BETWEEN TOTAL MOISTURE CONTENT INDEXES IN FROZEN ROCKS AND SOIL

Moisture in frozen rocks and soil is generally represented as two components – ice and unfrozen water. To quantify the moisture content in these rocks, two indexes called total and general moisture content were earlier proposed. The paper shows that such names do not properly reflect these indexes. In fact, it is the total mass moisture and water content. For a more complete and more accurate assessment of the impact of moisture on the physical properties of frozen rocks and soil, the paper proposes using total volumetric and absolute moisture and water content. The relation between total moisture indexes in frozen rocks and soil considered in the paper was identified.

Keywords: frozen rock, total moisture, total water content, total absolute moisture, total absolute water content, the relation between them.

To assess the content of all kinds of moisture (ice, frozen water) in frozen rocks and soil (hereafter frozen ground) two indexes, *total and general moisture* signified in the absolute majority of literature sources as $W_{t.}$ and $W_{g.}$ [1–6], were proposed. Total moisture was defined as the relation of moisture mass in frozen ground $M_m = M_{ice} + M_{nonfr.w}$, where M_{ice}, $M_{nonfr.w}$ is the mass, kg, ice and non-frozen water to mass of dry rocks and soil $M_{dry\ ground}$, i.e. to the mass of mineral matrix $M_{\min}$, and general moisture as relation of M_m to the mass of frozen ground M_{gr}.

However, it is necessary to note that the definitions proposed for $\mathrm{W}_{\mathrm{t.}}$ and $\mathrm{W}_{\mathrm{g.}}$ do not exactly correctly depict the essence of these indexes.

In the classic approach to determining moisture content in substances [7, 8], substance moisture means the moisture content in them, related to the wet condition of substances. If moisture content in a substance is related to its dry condition, this is already moisture content. Considering the fact that $\mathrm{W}_{\mathrm{t.}}$ is determined by the dry condition of rock, $\mathrm{W}_{\mathrm{t.}}$ will actually be *total mass water content* with a total dimension of "kg of moisture/kg of dry ground". It is better written as $w_{\Sigma,\mathrm{M}}$ (in technical literature water content indexes are usually indicated by lower case w with corresponding indexes. Here the lower index Σ is a conventional designation of sum, and m of mass). General moisture W_{g} is total mass moisture. It is better written as $W_{\Sigma,\mathrm{m}}$ (in technical literature moisture indexes are usually indicated by the capital letter W with corresponding indexes). Therefore, we have

$$W_{\Sigma,m} = \frac{M_m}{M_{fr.\,gr}} = \frac{M_{ice} + M_{nonfr.\,w}}{M_{fr.\,gr}};$$

$$w_{\Sigma,m} = \frac{M_m}{M_{dry.\,gr}} = \frac{M_{ice} + M_{nonfr.\,w}}{M_{dry.\,gr}}. \qquad (1)$$

Dimension $\left[W_{\Sigma,\text{м}}, w_{\Sigma,\text{м}}\right] = \text{fraction}$.

If we multiply the first parts of the fractions in (1) by 100, $W_{\Sigma,m}$ and $w_{\Sigma,m}$ is measured in percent by weight (% mass), irrespective of the dimensions stated for these indexes, *unit fractions* or *mass percent*, their total dimensions will be:

$\left[W_{\Sigma,\,m}\right] =$ kg of moisture/kg of frozen grounds;

$\left[w_{\Sigma,\,m}\right] =$ kg of moisture/kg of dry grounds.

So, according to the dimensional theory

$\left[M_{fr.\,gr}\right] =$ kg of frozen ground =
= kg of dry ground + kg of moisture

and this is irrespective of whether the ground is loosen or frozen.

However, as noted in [9], the use of only mass characteristics of water content does not allow for a full analysis of moisture impact on the corresponding physical properties of frozen ground. It would be more convenient and clear to use volumetric data of moisture content determined in certain cases like $W_{\Sigma,\,m}$ and $w_{\Sigma,\,m}$:

$$W_{\Sigma,\,vol}=\frac{V_m}{V_{fr.gr}}=\frac{V_m}{K_vV_{dry\,gr}}=\frac{V_{ice}+V_{nonfr.\,w}}{K_VV_{dry\,gr}};$$
$$w_{\Sigma,\,vol}=\frac{V_m}{K_fV_{dry\,gr}}=\frac{V_{ice}+V_{nonfr.\,w}}{K_fV_{dry\,gr}}, \qquad (2)$$

where $V_{fr.gr}$, $V_{dry\,gr}$ is volume, m^3, frozen (loosen and unripped) ground and dry unripped ground; V_{ice}, $V_{nonfr.\,w}$ is the volume of ice and non-frozen water in frozen ground, m^3; $K_f \geq 1.0$ is the fragmentation index of dry ground, unit fraction; K_V is the ratio of ground change as a result of fragmentation (destruction) and volumetric swell due to watering and following freezing of water, unit fraction.

The feature of the changing ground condition as a result of their watering and following the formation of ice is that, during watering ground, the water is located in pores, cracks and cavities changing the volume of ground directly due to an increase in their volume and indirectly impacting their swelling and rippability. The same happens when freezing water. It should also be taken into account that in wet ground especially in cohesive and loosen ground, the fragmentation index K_f and swelling index K_{sw} depend on their moisture content. This dependence for a number of rocks can sometimes be rather complex. For example, by watering fine-grained ground such as quartz sand, a decrease in K, as well as its increase can occur. This is established by the fact that at certain ranges moisture can facilitate the agglomeration of fine hard particles into larger conglomerates, and the other way round, at other ranges. K_f and K_{sw} change through even more complex dependences while water freezes, especially in cohesive ground where the ice formation process ends at temperatures of much lower than 0°C.

Therefore, a change in volume of dry ground only happens due to their fragmentation (destruction), while K_f at plus and minus temperatures is actually the same. Wet grounds in unripped (undistracted) condition, in that case $K_f = 1.0$, change their volume in the process of watering and following freezing of water. So, in these grounds $K_V = K_{sw}$. In friable grounds the process of swelling due to watering and following freezing of water is accompanied by the fragmentation process. So, their K_V becomes an integral index indicated in general as $K_V = K_f K_{sw}$. In practice, when indicating the changing volume of friable grounds as a result of changing their fragmentation, watering and freezing of water, it is very difficult to distinguish the impact of K_f and K_{sw} on K_V. So, in these grounds K_V is experimentally determined, also taking into account the impact of K_f and K_{sw}.

If in (2) you multiply the right parts of fractions by 100, $W_{\Sigma,V}$ and $w_{\Sigma,V}$ will be measured in percent by volume (%V). However, you should not forget that irrespective of the dimensions of these indexes, unit fractions or volume percent, their full dimensions for unripped or friable frozen grounds will be:

$\left[W_{\Sigma,v}\right]=$ m^3 moisture/m^3 frozen grounds = = m^3 moisture/m^3 dimension of frozen grounds;

$\left[w_{\Sigma,v}\right]=$ m^3 moisture/m^3 dry grounds = = m^3 moisture/m^3 dimension of dry grounds.

As well as mass and volume indexes of moisture, in many cases it is convenient to use indexes of mass concentration of moisture that can be assigned to frozen and dry ground. In the first case it will be total moisture $W_{\Sigma,\,\mathrm{abs}}$ of frozen ground, and in the second case it will be total absolute water content $w_{\Sigma,\,\mathrm{abs}}$ defined as:

$$W_{\Sigma,\,\mathrm{abs}}=\frac{M_m}{M_{fr.gr}}=\frac{M_{ice}+M_{nonfr.\,w}}{M_{fr.gr}}=$$
$$=\frac{M_{ice}+M_{nonfr.\,w}}{K_VV_{dry\,gr}};$$
$$w_{\Sigma,\,\mathrm{abs}}=\frac{M_m}{K_fV_{dry\,gr}}=\frac{M_{ice}+M_{nonfr.\,w}}{K_fV_{dry\,gr}}.$$

All indexes of moisture content in frozen grounds $W_{\Sigma,\, m}$, $W_{\Sigma,\, vol}$, $w_{\Sigma,\, m}$, $w_{\Sigma,\, vol}$, $W_{\Sigma,\, \mathrm{abs}}$ and $w_{\Sigma,\, \mathrm{abs}}$ are banded by pairs through parameters of ground condition and density properties of ground and moisture components. This makes it possible to determine other indexes, knowing some of them, and in some cases assess the condition parameters of frozen ground and density properties. However, even for one pair of indexes, the types of relation can differ. This is determined by the approach to determining them. This will be shown below for a number of cases. It is not difficult to find these relations, however, it is important to clearly understand the essence of the indexes used. Relations between $W_{\Sigma,\, m}$ and $w_{\Sigma,\, m}$ have the same appearance as those between total moisture W_m and total moisture content w_m in wet ground:

$$W_{\Sigma,\, m} = \frac{M_m}{M_{fr.\, gr}} = \frac{M_m}{M_{dry\ gr} + M_m} = \frac{M_m / M_{dry\ gr}}{\dfrac{M_{dry\ gr}}{M_{dry\ gr}} + \dfrac{M_m}{M_{dry\ gr}}} = \frac{w_{\Sigma,\, m}}{1 + w_{\Sigma,\, m}}.$$

The relation between $W_{\Sigma,\, m}$ and $W_{\Sigma,\, vol}$ has a more complicated appearance:

$$\frac{W_{\Sigma,\, m}}{W_{\Sigma,\, vol}} = \frac{M_m}{M_{fr.gr}} : \frac{V_m}{V_{fr.gr}} = \frac{M_m}{V_m} \cdot \frac{V_{fr.gr}}{M_{fr.gr}} = \rho_m \frac{K_f V_{dry\ gr}}{M_{dry\ gr} + M_m} = \rho_m \frac{K_V}{\dfrac{M_{dry\ gr}}{V_{dry\ gr}} + \dfrac{M_m}{V_{dry\ gr}}} =$$

$$= \rho_m \frac{K_V}{\rho_{vol} + \dfrac{\rho_{vol} M_m}{\rho_{vol} V_{dry\ gr}}} = \rho_m \frac{K_V}{\rho_{vol} + \dfrac{\rho_{vol} M_m}{M_{dry\ gr}}} = \rho_m \frac{K_V}{\rho_{vol} + \rho_{vol} w_{\Sigma,\, m}}. \quad (3)$$

This relation may be also obtained in another way. For example:

$$\frac{W_{\Sigma,\, \text{м}}}{W_{\Sigma,\, \text{об}}} = \rho_m \frac{K_V V_{dry\ gr}}{M_{dry\ gr} + M_m} = \rho_m \frac{K_V}{\dfrac{M_{dry\ gr}}{V_{dry\ gr}} + \dfrac{K_f V_m}{K_f V_{dry\ gr}}} = \rho_m \frac{K_V}{\rho_{vol} + K_f w_{\Sigma,\, \mathrm{abs}}}. \quad (4)$$

where ρ_m is moisture density in frozen ground, kg/m^3; ρ_{vol} is volumetric density, kg/m^3, of ground defined in accordance with the recommendations in the paper [10], $\rho_{vol} = M_{dry\ gr} / V_{dry\ gr} = \rho\left(1 - P_{v.d}\right)$; ρ – density, kg/m^3, of ground, $\rho = M_{dry\ gr} / V_{min.m} = M_{min.m} / V_{min.m}$, $V_{min.m}$ – volume of mineral matrix of ground, m^3; $P_{v.d} = V_{cav} / V_{dry\ gr}$ – index of total porosity, unit fraction; V_{cav} – total volume of all cavities in dry undisturbed (unripped) ground, m^3.

Moisture in frozen ground is a two-component substance consisting of unfrozen water and ice. Taking into account calculation methods of many-component substances density, we get

$$\rho_m = \rho_{ice} \frac{I_{\Sigma, v, ice}}{I_{\Sigma, v, ice} + W_{v, nonfr.w}} + \rho_w \frac{W_{v, nonfr.w}}{I_{\Sigma, v, ice} + W_{v, nonfr.w}} = \frac{1}{\dfrac{I_{\Sigma,\, m,\, ice}}{\left(I_{\Sigma,\, m,\, ice} + W_{m, nonfr.w}\right)\rho_{ice}} + \dfrac{W_{m, nonfr.w}}{\left(I_{\Sigma,\, m,\, ice} + W_{m\ nonfr.w}\right)\rho_w}}, \quad (5)$$

where $I_{\Sigma, m, ice} = I_{ice} / V_{fr.gr}$, $W_{v, nonfr.w} = V_{nonfr.w} / V_{fr.gr}$ is the total volumetric ice content of frozen ground, unit fraction, and its volumetric moisture due to unfrozen water, unit fraction; $I_{\Sigma, v, ice} = M_{ice} / M_{fr.gr}$, $W_{m, nonfr.w} = M_{nonfr.w} / M_{fr.gr}$ is the total mass ice content of frozen ground, unit fraction; and its moisture due to unfrozen water, unit fraction; ρ_{ice}, ρ_w – ice density, $\rho_{ice} \approx 917$ kg/m^3, and unfrozen water density, $\rho_w \approx 1000$ kg/m^3.

Taking into account values ρ_{ice}, ρ_w, let us present (5) as follows:

$$\rho_m = 917\frac{I_{\Sigma,v,ice}}{I_{\Sigma,v,ice}+W_{v,nonfr.w}} +$$

$$+1000\frac{W_{v,nonfr.w}}{I_{\Sigma,v,ice}+W_{v,nonfr.w}} \approx$$

$$\approx \frac{917 I_{\Sigma,v,ice} + 1000 W_{v,nonfr.w}}{I_{\Sigma,v,ice} + 1000 W_{v,nonfr.w}} \approx$$

$$\approx \frac{1}{\dfrac{0.0011 I_{\Sigma,m,ice}}{I_{\Sigma,m,ice}+W_{m,nonfr.w}} + \dfrac{0.001 W_{m,nonfr.w}}{I_{\Sigma,m,ice}+W_{m,nonfr.w}}}.$$

Let us check the obtained value expressions (3) and (4) using the dimension theory. Let’s consider two cases. The first one is when dry ground is unripped, i.e. $K_f = 1.0$ and the second is when it is loosen, i.e. $K_f > 1.0$. So, in the first case we take into account that $K_V = \dfrac{\text{m}^3 \text{ fr. gr}}{\text{m}^3 \text{ dry gr}}$.

The dimensions of the left part of value expressions (3) and (4) are

$$\left[\frac{W_{\Sigma,m}}{W_{\Sigma,v}}\right] = \left[\frac{M_m V_{fr.gr}}{V_m M_{fr.gr}}\right] =$$

$$= \frac{\text{kg moisture} \cdot \text{m frozen ground}}{\text{m}^3 \text{ moisture} \cdot \text{kg frozen ground}},$$

and of the right part of (3)

$$\left[\rho_m \frac{K_V}{\rho_{об} + \rho_{об} w_{\Sigma,\text{м}}}\right] = \frac{\text{kg moisture}}{\text{m}^3 \text{ moisture}} \times \frac{\text{m}^3 \text{ fr. gr.}}{\text{m}^3 \text{ dry gr.}} \times$$

$$\times \frac{1}{\left(\dfrac{\text{kg dry gr.}}{\text{m}^3 \text{ dry gr.}} + \dfrac{\text{kg dry gr.}}{\text{m}^3 \text{ dry gr.}} \cdot \dfrac{\text{kg moisture}}{\text{kg dry gr.}}\right)} =$$

$$= \frac{\text{kg moisture} \cdot \text{m}^3 \text{ fr. gr.}}{\text{m}^3 \text{ moisture} \cdot \text{m}^3 \text{ dry gr.} \left(\dfrac{\text{kg dry gr.} + \text{kg moisture}}{\text{m}^3 \text{ dry gr.}}\right)} =$$

$$= \frac{\text{kg moisture} \cdot \text{m}^3 \text{ fr. gr.}}{\text{m}^3 \text{ moisture} \cdot \text{kg}^3 \text{ fr. gr.}}.$$

Comparing the dimensions of both parts of value expression (3), we can see that they are equal. Exactly the same conclusion is obtained by consideration of value expression (4).

Let us consider the second case for value expression (4). Dimensions of K_V and K_f and the left part of value expression (4) are:

$$[K_V] = \frac{\text{m}^3 \text{ loosen fr. gr.}}{\text{m}^3 \text{ dry gr.}};$$

$$\left[\frac{W_{\Sigma,m}}{W_{\Sigma,vol}}\right] = \left[\frac{M_m V_{\text{loosen fr. gr}}}{V_m \text{M}_{\text{fr. gr}}}\right] =$$

$$= \frac{\text{kg moisture} \cdot \text{m}^3 \text{loosen fr. gr}}{\text{m}^3 \text{moisture} \cdot \text{kg fr. gr}},$$

where $V_{\text{loosen fr. gr}}$ is the volume of loosen frozen ground, m^3.

When analyzing the left part dimensions of value expressions (3) and (4), it is considered that in the second case the same total mass moisture of loosen and unripped frozen grounds mass of loosen frozen ground equals to a mass of unripped frozen ground, since cavities between separate sections of frozen ground as a result of its fragmentation at $W_{\Sigma,m} = \text{const}$ mass of the ground is not changed. However, one should also take into account that $V_{\text{loosen fr. gr}} \neq V_{\text{fr. gr}}$.

For the right part of (4) we have:

$$[\rho_{вл} K_V] = \frac{\text{kg moisture}}{\text{m}^3 \text{ moisture}} \cdot \frac{\text{m}^3 \text{ loosen fr. gr}}{\text{m}^3 \text{ dry gr}};$$

$$\left[\rho_{vol} + K_f w_{\Sigma,\text{м}}\right] = \frac{\text{kg dry gr.}}{\text{m}^3 \text{ dry gr.}} + \frac{\cancel{\text{m}^3 \text{ dry loosen gr.}}}{\text{m}^3 \text{ dry gr.}} \times$$

$$\times\frac{\text{kg moisture}}{\cancel{\text{m}^3\ \text{dry loosen gr.}}}=\frac{\text{kg dry gr.}}{\text{m}^3\ \text{dry gr.}}+\frac{\text{kg moisture}}{\text{m}^3\ \text{dry gr.}}=$$

$$=\frac{\text{kg fr. gr.}}{\text{m}^3\ \text{dry gr.}}.$$

Finally:

$$\left[\rho_m\frac{K_V}{\rho_{vol}+K_f w_{\Sigma,\,m}}\right]=\rho_m K_V:\left[\rho_{vol}+K_f w_{\Sigma,\,m}\right]=$$

$$=\frac{\text{kg moisture}\cdot\text{m}^3\ \text{loosen fr. gr.}}{\text{m}^3\ \text{moisture}\cdot\text{m}^3\ \text{dry gr.}}:\frac{\text{kg fr. gr.}}{\text{m}^3\ \text{dry gr.}}=$$

$$=\frac{\text{kg moisture}\cdot\text{m}^3\ \text{loosen fr. gr.}}{\text{m}^3\ \text{moisture}\cdot\text{kg fr. gr.}}.$$

Therefore, the dimensions of the left and right parts of (4) are the same. Exactly the same result is obtained by consideration of value expression (3) for loosen frozen grounds.

The relations between $W_{\Sigma,m}$ and $w_{\Sigma,vol}$ have the same appearance as the relations of (3) and (4) between $W_{\Sigma,m}$ and $W_{\Sigma,vol}$:

$$\frac{W_{\Sigma,\,m}}{w_{\Sigma,\,vol}}=\frac{M_m}{M_{fr.\,gr}}:\frac{V_m}{K_f V_{dry\ gr}}=$$

$$=\frac{M_m}{V_m}\cdot\frac{K_f V_{dry\ gr}}{M_{fr.\,gr}}=\rho_m\frac{K_f V_{dry\ gr}}{M_{dry\ gr}+M_m}$$

$$=\rho_m\frac{K_f}{\dfrac{M_{dry\ gr}}{V_{dry\ gr}}+\dfrac{M_m}{V_{dry\ gr}}}=\rho_m\frac{\text{К}_\text{р}}{\rho_{vol}+\dfrac{\rho_{vol}M_m}{\rho_{vol}V_{dry\ gr}}}=$$

$$=\rho_m\frac{K_f}{\rho_{vol}+\dfrac{\rho_{vol}M_m}{M_{dry\ gr}}}=\rho_m\frac{K_f}{\rho_{vol}+\rho_{vol}w_{\Sigma,\,m}};$$

$$\frac{W_{\Sigma,m}}{w_{\Sigma,vol}}=\rho_m\frac{K_f}{\dfrac{\text{M}_{dry\,gr}}{V_{dry\,gr}}+\dfrac{\text{M}_m}{V_{dry\,gr}}}=$$

$$=\rho_m\frac{K_f}{\rho_{vol}+\dfrac{K_f\text{M}_{\text{вл}}}{K_f V_{dry\,gr}}}=\rho_m\frac{K_f}{\rho_{vol}+K_f w_{\Sigma,\text{abs}}}.$$

Let’s find the relation between $W_{\Sigma,m}$ and $W_{\Sigma,\text{abs}}$:

$$\frac{W_{\Sigma,m}}{W_{\Sigma,\text{abs}}}=\frac{M_m}{M_{fr.gr}}:\frac{M_m}{V_{fr.gr}}=\frac{V_{fr.gr}}{M_{fr.gr}}=\frac{K_V V_{dry\,gr}}{\text{M}_{dry\,gr}+M_m}=$$

$$=\frac{K_V}{\dfrac{M_{dry\,gr}}{V_{dry\,gr}}+\dfrac{M_m}{V_{dry\,gr}}}=\frac{K_V}{\rho_{vol}+\dfrac{\rho_{vol}M_m}{\rho_{vol}V_{dry\,gr}}}=$$

$$=\frac{K_V}{\rho_{vol}+\dfrac{\rho_{vol}M_m}{M_{dry\,gr}}}=\frac{K_V}{\rho_{vol}\left(1+w_{\Sigma,\text{м}}\right)}. \qquad (6)$$

In unripped frozen grounds ($K_f=1.0$) the relation (6) has the following appearance:

$$\frac{W_{\Sigma,m}}{W_{\Sigma,\text{abs}}}=\frac{K_{sw}}{\rho_{vol}\left(1+w_{\Sigma,m}\right)}.$$

If we use volumetric density of frozen ground $\rho_{vol,fr.gr}$ and pour density of frozen ground $\rho_{pour,fr.gr}$, the relation between $W_{\Sigma,m}$ and $W_{\Sigma,\text{abs}}$ can be presented as follows:

in unripped frozen ground

$$\frac{W_{\Sigma,m}}{W_{\Sigma,\text{abs}}}=\frac{V_{fr.gr}}{M_{fr.gr}}=\frac{1}{\rho_{vol,\,fr.gr}},$$

in loosen ground

$$\frac{W_{\Sigma,m}}{W_{\Sigma,vol}}=\frac{V_{fr.gr}}{M_{fr.gr}}=\frac{1}{\rho_{pour,\,fr.gr}}.$$

One of the types of relation between $W_{\Sigma,m}$ and $W_{\Sigma,\text{abs}}$ can be obtained as follows:

$$\frac{W_{\Sigma,m}}{w_{\Sigma,\text{abs}}}=\frac{M_m}{\text{M}_{fr.gr}}:\frac{M_m}{K_f V_{dry\,gr}}=\frac{K_f V_{dry\,gr}}{\text{M}_{fr.gr}}=$$

$$=\frac{K_f V_{dry\,gr}}{\text{M}_{dry\,gr}+M_m}=\frac{K_f}{\dfrac{\text{M}_{dry\,gr}}{V_{dry\,gr}}+\dfrac{M_m}{V_{dry\,gr}}}=$$

$$=\frac{K_f}{\rho_{vol}+\dfrac{\rho_{vol}M_m}{\rho_{vol}V_{dry\,gr}}}=\frac{K_f}{\rho_{vol}+\dfrac{\rho_{vol}M_m}{\text{M}_{dry\,gr}}}=$$

$$= \frac{K_f}{\rho_{vol}\left(1 + w_{\Sigma,m}\right)}. \qquad (7)$$

Using another approach to search for a relation between $W_{\Sigma,m}$ and $W_{\Sigma,\text{abs}}$, we get:

$$\frac{W_{\Sigma,m}}{w_{\Sigma,\text{abs}}} = \frac{M_m K_f V_{dry\ fr.}}{M_{fr.\ gr} M_m} = \frac{K_f V_{dry\ fr.}}{M_{fr.\ gr}} =$$

$$= \frac{K_f V_{dry\ fr.}}{M_{dry\ fr.} + M_m} = \frac{K_f}{\dfrac{M_{dry\ fr.}}{V_{dry\ fr.}} + \dfrac{M_m}{V_{dry\ fr.}}} =$$

$$= \frac{K_f}{\rho_{vol} + \dfrac{K_f M_m}{K_f V_{dry\ fr.}}} = \frac{K_f}{\rho_{vol} + K_f w_{\Sigma,\text{аб}}}; \rightarrow$$

$$\rightarrow \frac{W_{\Sigma,m}}{w_{\Sigma,\text{abs}}} = \frac{K_f}{\rho_{vol} + K_f w_{\Sigma,\text{аб}}}; \rightarrow$$

$$\rightarrow W_{\Sigma,m} = \frac{w_{\Sigma,\text{abs}} K_f}{\rho_{vol} + K_f w_{\Sigma,\text{abs}}}. \qquad (8)$$

In unripped frozen grounds ($K_f = 1.0$) both relations (7) and (8) between $W_{\Sigma,m}$ and $W_{\Sigma,\text{abs}}$ have the following appearance:

$$\frac{W_{\Sigma,m}}{w_{\Sigma,\text{abs}}} = \frac{1}{\rho_{\text{об}}\left(1 + w_{\Sigma,m}\right)};$$

and

$$W_{\Sigma,m} = \frac{w_{\Sigma,\text{abs}}}{\rho_{vol} + w_{\Sigma,\text{abs}}}.$$

By analogy, let us find consistent relations between other total indexes of moisture content in frozen ground that were not considered above $W_{\Sigma,vol}$ and $W_{\Sigma,m}$, $W_{\Sigma,vol}$ and $w_{\Sigma,vol}$, $W_{\Sigma,vol}$ and $W_{\Sigma,\text{abs}}$, $W_{\Sigma,vol}$ and $w_{\Sigma,\text{abs}}$, $W_{\Sigma,\text{abs}}$ and $w_{\Sigma,vol}$, $W_{\Sigma,\text{abs}}$ and $w_{\Sigma,\text{abs}}$, $w_{\Sigma,m}$ and $w_{\Sigma,vol}$, $w_{\Sigma,m}$ and $w_{\Sigma,\text{abs}}$, $w_{\Sigma,vol}$ and $w_{\Sigma,\text{abs}}$:

$$\frac{W_{\Sigma,vol}}{w_{\Sigma,m}} = \frac{V_m}{V_{fr.\ gr}} : \frac{M_m}{\mathrm{M}_{dry\ gr}} = \frac{V_m \mathrm{M}_{dry\ gr}}{V_{fr.\ gr}\mathrm{M}_{\text{вл}}} =$$

$$= \frac{V_m}{M_m} \times \frac{\mathrm{M}_{dry\ gr}}{\mathrm{K}_{\mathrm{v}} V_{dry\ gr}} = \frac{1}{\rho_m} \times \frac{\rho_{vol}}{\mathrm{K}_V} = \frac{\rho_{vol}}{\mathrm{K}_V \rho_m},$$

$$\frac{W_{\Sigma,vol}}{w_{\Sigma,vol}} = \frac{V_m}{V_{fr.\ gr}} : \frac{V_m}{K_f V_{dry\ gr}} =$$

$$= \frac{V_m}{K_V V_{dry\ gr}} : \frac{V_m}{K_f V_{dry\ gr}} = \frac{K_f}{\mathrm{K}_V}, \qquad (9)$$

if frozen ground is not loosen, i.e. $K_f = 1.0$, then in this case (9) transforms into (10):

$$\frac{W_{\Sigma,vol}}{w_{\Sigma,vol}} = \frac{1}{K_{sw}}; \qquad (10)$$

$$\frac{W_{\Sigma,vol}}{W_{\Sigma,\text{abs}}} = \frac{V_m}{V_{fr.\ gr}} : \frac{\mathrm{M}_m}{V_{fr.\ gr}} = \frac{V_m}{\mathrm{M}_m} = \frac{1}{\rho_m};$$

$$\frac{W_{\Sigma,vol}}{w_{\Sigma,\text{abs}}} = \frac{V_m}{V_{fr.\ gr}} : \frac{M_m}{K_f V_{dry\ gr}} =$$

$$= \frac{V_m}{M_m} \times \frac{K_f V_{dry\ gr}}{K_V V_{dry\ gr}} = \frac{K_f}{K_V \rho_m};$$

$$\frac{W_{\Sigma,\text{abs}}}{w_{\Sigma,vol}} = \frac{M_m}{V_{fr.\ gr}} : \frac{V_m}{K_f V_{dry\ gr}} =$$

$$= \frac{M_m}{V_m} \times \frac{K_f V_{dry\ gr}}{K_V V_{dry\ gr}} = \rho_m \frac{K_f}{K_V};$$

$$\frac{W_{\Sigma,\text{abs}}}{w_{\Sigma,\text{abs}}} = \frac{M_m}{K_V V_{\text{с. пор}}} : \frac{M_m}{K_f V_{\text{с. пор}}} = \frac{K_f}{\mathrm{K}_V};$$

$$\frac{w_{\Sigma,m}}{w_{\Sigma,vol}} = \frac{M_m}{M_{dry\ gr}} : \frac{V_m}{K_f V_{\text{с. пор}}} =$$

$$= \frac{M_m}{V_m} \times \frac{K_f V_{dry\ gr}}{M_{dry\ gr}} = \rho_m \frac{K_f}{\rho_{\text{об}}};$$

$$\frac{w_{\Sigma,m}}{w_{\Sigma,\text{abs}}} = \frac{M_m}{M_{dry\ gr}} : \frac{M_m}{K_f V_{dry\ gr}} = \frac{K_f V_{dry\ gr}}{M_{dry\ gr}} = \frac{K_f}{\rho_{vol}};$$

$$\frac{w_{\Sigma,vol}}{w_{\Sigma,\text{abs}}} = \frac{V_m}{K_f V_{dry\ gr}} : \frac{M_m}{K_f V_{dry\ gr}} = \frac{V_m}{M_m} = \frac{1}{\rho_m}.$$

The performed research allowed us to determine for the first time the type of relation between indexes of ice and unfrozen water content in frozen ground and the total moisture indexes in these grounds. It has been shown that, in a number of cases, the relation between the same indexes can differ, which is caused by their determination method. Using the obtained indexes in practice makes it possible to reduce the number of laboratory studies for determining the relations between other corresponding indexes of physical properties of frozen ground and total moisture content indexes and certain components in this ground.

References

1. Soil science: Classical university textbook / V.T. Trofimov, V.A. Korolev, Y.A. Voznesenskiy and others; edited by V.T. Trofimov. – Edition 6, revised and amended – M.: Publishing house of MSU, 2005. – 1024 p.
2. Tsytovich N.A. Mechanics of frozen soils (general and applied): Textbook for universities. – M.: High school, 1973. – 448 p.
3. Guide for construction on permafrost / A.F. Antonov, Y.A. Velly, V.V. Galperin and others; Edited by Y.A. Velly, V.I. Doluchayev, N.F. Fedorov. – L.: Stroyizdat, Leningrad department, 1977. – 552 p.
4. Anderson D.M., Pusch R., Penner E. Physical and thermal properties of frozen ground // Geotechnical Engineering for Cold Regions (O.B. Andersland and D.M.Anderson, Eds). N. Y.: Mc. Craw-Hill. 1978. – P. 37–102.
5. Fletcher N.H. The chemical of ice. Cambridge Univ. Press. 1970. – 271 p.
6. Hoekstra P. The physics and chemistry of frozen soils // Highway Research Board. Spec. Rep. № 103, 1969. – P. 78–80.
7. Great Soviet Encyclopedia. – M.: Soviet Encyclopedia. Vol. 5. Veshin-gazli, 1971. – 640 p.
8. Korneyeva T.V. Defining dictionary of metrology, measuring equipment and quality management. Key terms: about 7,000 terms. – M.: Russian language, 1990. – 464 p.
9. Frolov A.D. Electrical and elastic properties of frozen rocks and ice. – Pushchino: Dept. of Scientific and Technical Information of Pushchino Scientific Center of Russian Academy of Sciences, 1998. – 515 p.
10. Yanchenko G.A. On the density indexes of rocks and minerals // College notice. Mining magazine. – 2007. – No. 4. – p. 139-149.

“Gornye nauki i tehnologii”/ “Mining science and technology”, 2016, No. 1, pp. 25-31	
Title:	**The relation between total moisture content indexes in frozen rocks and soil**
Author 1	Name&Surname: **Gennady A. Yanchenko** Company: **National University of Science and Technology NUST MISiS** Work Position: **Professor**
DOI:	**http://dx.doi.org/10.17073/2500-0632-2016-1-25-32**
Abstract:	Moisture in frozen rocks and soil is generally represented as two components – ice and unfrozen water. To quantify the moisture content in these rocks, two indexes called total and general moisture content were earlier proposed. The paper shows that such names do not properly reflect these indexes. In fact, it is the total mass moisture and water content. For a more complete and more accurate assessment of the impact of moisture on the physical properties of frozen rocks and soil, the paper proposes using total volumetric and absolute moisture and water content. The relation between total moisture indexes in frozen rocks and soil considered in the paper was identified.
Keywords:	frozen rock, total moisture, total water content, total absolute moisture, total absolute water content, the relation between them.
References:	1. Soil science: Classical university textbook / V.T. Trofimov, V.A. Korolev, Y.A. Voznesenskiy and others; edited by V.T. Trofimov. – Edition 6, revised and amended – M.: Publishing house of MSU, 2005. – 1024 p. 2. Tsytovich N.A. Mechanics of frozen soils (general and applied): Textbook for universities. – M.: High school, 1973. – 448 p. 3. Guide for construction on permafrost / A.F. Antonov, Y.A. Velly, V.V. Galperin and others; Edited by Y.A. Velly, V.I. Doluchayev, N.F. Fedorov. – L.: Stroyizdat, Leningrad department, 1977. – 552 p. 4. Anderson D.M., Pusch R., Penner E. Physical and thermal properties of frozen ground // Geotechnical Engineering for Cold Regions (O.B. Andersland and D.M.Anderson, Eds). N. Y.: Mc. Craw-Hill. 1978. – P. 37–102.

5. Fletcher N.H. The chemical of ice. Cambridge Univ. Press. 1970. – 271 p.
6. Hoekstra P. The physics and chemistry of frozen soils // Highway Research Board. Spec. Rep. № 103, 1969. – P. 78–80.
7. Great Soviet Encyclopedia. – M.: Soviet Encyclopedia. Vol. 5. Veshin-gazli, 1971. – 640 p.
8. Korneyeva T.V. Defining dictionary of metrology, measuring equipment and quality management. Key terms: about 7,000 terms. – M.: Russian language, 1990. – 464 p.
9. Frolov A.D. Electrical and elastic properties of frozen rocks and ice. – Pushchino: Dept. of Scientific and Technical Information of Pushchino Scientific Center of Russian Academy of Sciences, 1998. – 515 p.
10. Yanchenko G.A. On the density indexes of rocks and minerals // College notice. Mining magazine. – 2007. – No. 4. – p. 139-149.