THE CALCULATION OF MULTILAYER LINING OF TUNNELS, CONSTRUCTED IN A TECHNOLOGICALLY DIVERSE OF MASSIF SOIL

Addressing urban transport is a very timely matter, especially in the capital Hanoi and Ho Chi Minh City. In order to solve this problem, a solution has been proposed for the construction of overhead tram and subway lines. In fact, when constructing subway lines through historical sites, high population density, many surface structures, etc., the method of open construction is not feasible, it is necessary to use the method Underground construction. These areas are often weak soil, the physical parameters of the soil detrimental to the tunnel construction work; Such as small stickiness, small internal friction angle, high porosity, high permeability coefficient, high water saturation, short shear strength etc. These factors create complex geological conditions in Construction tunnel. With that in mind, the calculation of the selection of the tunnel casing structure is necessary, which is timely.

This paper provides a solution to the problem of stress state of multilayer lining supporting the tunnel of circular cross-section, constructed in a technologically heterogeneous array. The tunnel lining and surrounding soil mass are considered as elements of a united deformable system.

Keywords: soil mass, technological heterogeneity, tunnel lining, stress, strain, elasticity theory, calculation.

1. Introduction

The task of calculating multilayer tunnel lining of circular cross-section, constructed in soils simulated by a homogeneous elastic medium, is solved by many authors [1, 2, 4, 9, 15, 17–20]. However, rigorous solutions to the problem of stress state of multilayer lining of a tunnel, constructed in a technologically diverse array of soil, so far not available [8, 10–14, 16].

2. Basic theoretical principles

In the task, the calculated diagram is shown in Fig. 1, multilayer concentric ring consisting of arbitrary number of layers, the boundaries of which consists of a circle with the center placed at the origin.

Fig. 1. A design scheme for a multilayer lining of a tunnel, constructed in a technologically heterogeneous array

Here the outer layer of the ring $S_1$ infinitely large thickness (this is achieved if we put $R_0 \to \infty$ ) models the soil mass in its natural state. The portion of the stack $S_i$ ( $i = 2,3,\ldots,N^* $ ) models the area of technological heterogeneity of the array. The inner layers $S_j$ ( $ j = N^*+1,N^*+2,\ldots,N $ )
model the tunnel lining. The inner radii of the layers marked with \( R_i \) (\( i = 1, ..., N \)).

It is believed that the material of each layer of rings has its own, different in the General case, the deformation characteristics of \( E_i, \mu_i \) (\( i = 1, ..., N \)) be the moduli of deformation and Poisson's ratios respectively. The layers in the system to deform together, i.e. at the lines of contact of the layers \( L_i \) (\( i = 1, ..., N-1 \)) the conditions the irregularities of the displacements and full stresses.

External \( L_0 \) outer and inner contours \( L_N \) ring is free from external forces.

The gravitational force in the layer (\( i = 1, 2, ..., N^{*} \)), which simulates the soil body (as in a natural state and exposed to the technological impact of the excavation of the tunnel), is modeled by the presence of the same component fields of initial stresses \( \sigma_{x}^{(0)(i)} = \sigma_{y}^{(0)(i)} = -\gamma H \alpha \) (\( i = 1, 2, ..., N^{*} \))

Imagine full voltage in each layer of \( S_i \) (\( i = 1, 2, ..., N \)) in the form
\[
\sigma^{(i)} = \sigma^{(i)(i)} + \delta_{i,N^{*}+1} \sigma^{(0)(i)},
\]
e.g., charge amount and initial stress. In the expression (2) symbol \( \sigma \) marked all components of stresses, as a function of \( \delta_{n,m} \) is defined by the following expression
\[
\delta_{n,m} = \begin{cases} 1, & \text{if } n < m, \\ 0, & \text{if } n \geq m. \end{cases}
\]

We write the boundary conditions at the contact lines \( L_i \) (\( i = 1, 2, ..., N \)) rings through the additional stresses and displacements in the General form:
\[
\sigma_{r}^{(i)(i+1)} = \sigma_{r}^{(i)(i)} + \lambda_{i,N^{*}} \sigma_{r}^{(0)(i)}; \quad u_{i+1} = u_i.
\]

where functions \( \lambda_{n,m} \) (\( n = 1, 2, ..., N; \quad m = 1, 2, ..., N \)) are determined by the expression
\[
\lambda_{n,m} = \begin{cases} 1, & \text{if } n = m, \\ 0, & \text{if } n \neq m. \end{cases}
\]

Use the additional views radial contact stresses as the relevant pressures and introducing new symbols highlight from lining of any two adjacent layers, numbered respectively \( i \) and \( i+1 \) (\( i = 1, ..., N-1 \)), contact common edge \( L_i \) (Fig. 2).

Use the additional views radial contact stresses as the relevant pressures and introducing new symbols highlight the bolting of two arbitrary adjacent layers, numbered respectively \( i \) and \( i+1 \) (\( i = 1, ..., N-1 \)), in contact at the common boundary of \( L_i \) (Fig. 2).
When considering the equilibrium of selected layers, the effect of the discarded layers will replace the normal pressures. On the outer contour of \( L_{i-1} \) has pressure \( p_{i-1} \), simulating the effect of the discarded layers of \( S_i \) \((i=1, \ldots, i-1)\). The effect of the discarded inner layers \( S_j \) \((j=i+1, \ldots, N)\) is modeled by a uniform pressure \( p_{i+1} \), distributed according to the internal contour \( L_{i+1} \).

The displacements of points of the contours taking into account the fact that the outer circuit \( L_{i-1} \) each \( i \)-layer \( S_i \) \((i=1, \ldots, N)\) in the general case of a loaded pressure \( p_{i+1}+q_{i-1} \), take the form \([3-7]\).

Applying formula (6), should be taken into consideration that: \( p_0=q_0=p_N=q_N=0 \). (7)

From the second condition (4), reflecting the consistency of the deformations of two adjacent strata \( S_i \) and \( S_{i+1} \) in line contact \( L_i \), on the basis of the representations (6) have

\[
u_{q_i} = \frac{1}{2G_i} \left\{ (1-2\mu_i) \frac{p_i R_i^2 - (p_i+q_i) R_{i+1}^2}{R_i^2 R_{i+1}^2} R_i + \frac{(p_i - p_{i-1} - q_{i-1}) R_{i+1}^2}{R_{i+1}^2 - R_i^2} \right\},
\]

\[
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\]

Then convert the expression (8) can be written as:

\[
(1-2\mu_i) \frac{p_i c_i^2 - (p_{i-1} + q_{i-1})}{1-c_i^2} + \frac{(p_i - p_{i-1} - q_{i-1})}{1-c_i^2} =
\]

\[
= \frac{G_i}{G_{i+1}(1-c_{i+1}^2)} \left\{ (1-2\mu_i) \frac{p_{i+1} c_{i+1}^2 - (p_i + q_i) R_{i+1}^2}{1-c_{i+1}^2} R_i + \frac{(p_i - p_{i-1} - q_{i-1}) c_{i+1}^2}{1-c_{i+1}^2} \right\}.
\]

\[
c_i = \frac{R_i}{R_{i+1}} (i = 1, \ldots, N - 1).
\]

Imagine the resulting equality in the form

\[
\frac{G_i}{G_{i+1}(1-c_{i+1}^2)} \left[ 2(1-\mu_{i+1}) c_{i+1}^2 p_{i+1} + c_{i+1}^2 (p_i + q_i) \right] = \frac{1}{1-c_i^2} \left[ (1-2c_i^2 \mu_i + c_i^2) p_i - 2(1-\mu_i) (p_{i+1} + q_{i+1}) \right].
\]

Or

\[
\frac{G_i}{G_{i+1}(1-c_{i+1}^2)} \left[ 2(1-\mu_{i+1}) c_{i+1}^2 p_{i+1} = \frac{G_i}{G_{i+1}(1-c_{i+1}^2)} (1-2\mu_{i+1} + c_{i+1}^2) \times (p_i + q_i) + \frac{1}{1-c_i^2} (1-2c_i^2 \mu_i + c_i^2) p_i - \frac{2}{1-c_i^2} (1-\mu_i) (p_{i+1} + q_{i+1}) \right].
\]

From the last expression one can write
The resulting expression can be represented as:

\[ p_{i+1} = K_{i,i} p_i + K_{i,i-1} p_{i-1} + Q_i. \]  

The formula to determine the coefficients \( K_{i,i} \), \( K_{i,i-1} \) and free members \( Q_i \) can be obtained by comparing expressions (10) and (11):

\[
K_{i,i} = \frac{1}{2(1-\mu_{i+1})} \left[ \frac{1}{\xi_i} \left( 1 - 2c_i^2 \mu_i + c_i^2 \right) + 1 - 2c_i^2 \mu_{i+1} + c_i^2 \right];
\]

\[
K_{i,i-1} = \frac{(1-\mu_i)}{\xi_i (1-\mu_{i+1})} c_i^2; Q_i = \frac{1}{2(1-\mu_{i+1})} c_i^2 q_i - \frac{G_{i,i+1}}{G_i} \left( 1 - \frac{1}{\mu_{i+1}} \right) c_{i+1}^2 q_i. 
\]

where \( \xi_i = \frac{G_i}{G_{i+1}} \frac{1-c_i^2}{1-c_{i+1}^2}. \)

The recurrence formula (11) with \( i = 1 \) can be written as the ratio

\[ p_2 = K_{1,1} p_1 + K_{1,0} p_0 + Q_1, \]

which, taking into account the condition (7), is converted to the form:

\[ p_2 = K_{1,1} p_1 + Q_1 = M_2 p_1 + L_2, \]

where we have used the notation:

\[ M_2 = K_{1,1}; L_2 = Q_1. \]  

Further, using the representation (14), (15), applying the recurrence formula (11), putting \( i = 2 \), we can write:

\[ p_3 = K_{2,2} p_2 + K_{2,1} p_1 + Q_2 = K_{2,2} (M_2 p_1 + L_2) + K_{2,1} p_1 + Q_2 = M_3 p_1 + Q_3, \]

where:

\[ M_3 = K_{2,2} M_2 + K_{2,1}, L_3 = K_{2,2} L_2 + Q_2. \]

Continue reasoning similarly for \( i = 3, 4, \ldots, N-1 \).

So, when \( i = 3 \) we arrive to the expression:

\[ p_4 = K_{3,3} p_3 + K_{3,2} p_2 + Q_3 = K_{3,3} (M_3 p_1 + L_2) + K_{3,2} (M_2 p_1 + L_1) + Q_2 = M_4 p_1 + L_4, \]

where:

\[ M_4 = K_{3,3} M_3 + K_{3,2} M_2, \]

\[ L_4 = K_{3,3} L_3 + K_{3,2} L_2 + Q_2. \]

As a result, generalizing the representation (13)–(19) can write a General formula that allows to express all the unknown values \( p_i \) through \( p_1 \) in the form:

\[ p_i = M_i p_1 + L_i, \quad (i = 2, \ldots, N), \]

where:

\[ M_i = \sum_{j=2}^{i} K_{j-1,j} M_j; L_i = \sum_{j=2}^{i} K_{j-1,j} L_j + Q_{i-2}. \]

If take into account the representation (15) and (12), formulas (21) completely determine included in the expression (20) the values of \( M_i, L_i \) \((i = 2, \ldots, N)\).

In turn, the relation (20) allows at \( i=\text{N} \) to come to expression

\[ p_N = M_N p_1 + L_N. \]

On the other hand, based on (15) we have:

\[ p_N = M_N p_1 + L_N = 0. \]

Where will get:

\[ p_1 = -\frac{L_N}{M_N}. \]

Thus, substituting (24) into formula (20) allows to calculate all unknown values \( p_i \) \((i = 2, \ldots, N)\).

Full radial tension on the outer \( L_{i-1} \) and \( L_i \) the internal contours of the \( i \)-layer are defined by the formulas:

\[ \sigma_i^{(i-1)} |_{L_{i-1}} = p_{i-1} - \delta_{i,N-1+1} \gamma H a^+ \]

\[ \sigma_i^{(i)} |_{L_i} = p_i - \delta_{i,N-1+1} \gamma H a^+. \]

Normal tangential (circumferential) on these contours, respectively by the formulas...
\[ \sigma_{0}^{(i,j-1)} = \frac{2c_{r}^{2}}{c_{r}^{2} + 1} \sigma_{r}^{(i,j-1)} - \frac{c_{r}^{2} + 1}{c_{r}^{2} + 1} \sigma_{i}^{(j,i)} \quad \sigma_{0}^{(i,j)} = \frac{2}{1 - c_{r}^{2}} \sigma_{r}^{(i,j-1)} . \] (26)

The ability to determine the stresses in the layers of the underground constructions allow to verify the strength of concrete layer and the bearing capacity of underground construction in General formula [3]:

\[ N = \frac{\sigma_{0}^{(m)} + \sigma_{0}^{(ex)}}{2} \Delta b; \quad M = \frac{\sigma_{0}^{(m)} - \sigma_{0}^{(ex)}}{12} b \Delta^{2} , \] (27)

where \( \Delta = R_{0} - R_{1} = R_{0} \left( 1 - \frac{R_{1}}{R_{0}} \right) \) – thickness of the lining, \( b = 1 \) m.

Finally the formulas (27) taking

\[ N = \frac{1}{2} \left\{ \frac{2}{1 - \frac{R_{1}^{2}}{R_{0}^{2}}} q_{0} + \frac{1 + \frac{R_{1}^{2}}{R_{0}^{2}}}{1 - \frac{R_{1}^{2}}{R_{0}^{2}}} q_{0} \right\} b \left( R_{0} - R_{1} \right) = - \frac{q_{0}}{2} \frac{3 + \frac{R_{1}^{2}}{R_{0}^{2}}}{1 - \frac{R_{1}^{2}}{R_{0}^{2}}} b \left( 1 - \frac{R_{1}}{R_{0}} \right) = - \frac{q_{0}}{2} \frac{3 + \frac{R_{1}^{2}}{R_{0}^{2}}}{1 - \frac{R_{1}^{2}}{R_{0}^{2}}} \left( 1 - \frac{R_{1}}{R_{0}} \right) \left( 1 - \frac{R_{1}}{R_{0}} \right) \] (28)

The carrying capacity is estimated by the ratio:

\[ |N| \leq NS , \] (29)

where \( N \) – is the calculated normal force is determined from the first expression (28); \( NS \) – ultimate bearing capacity of the radial cross section of the lining defined by the ratio \( NS = k R_{b} \Delta b \left( 1 - \frac{2\varepsilon_{0}}{\Delta} \right) \), \( k = 1; \ \varepsilon_{0} = \left| \frac{M}{N} \right| \) – eccentricity of application of longitudinal force.

3. The calculation algorithm

Thus, the stress-strain state of layered underground structures, constructed in a technologically heterogeneous array, based on the study of the equilibrium state of a single deformable "multi-layered lining-array" represents the following sequence of operations:

1. The initial data are given by: \( N^* \) – the number of layers, modeling of technologically heterogeneous array of species; \( N \) -total number of layers modeling the system "multi-layered lining-rock mass" as a whole; \( R_{i} \) (i = 1, ..., \( N \)) are the radii of the layers of the lining (m); \( E_{i}, \mu_{i} \) – deformation characteristics of rock mass in its natural state - the deformation modulus (MPa) and Poisson's ratio respectively; \( E_{i}, \mu_{i} \) (i = 2, ..., \( N \)) be the modulus of deformation (MPa) and the Poisson's ratios of the material layers, modeling of technologically heterogeneous array of species (i = 2, 3, ..., \( N^* \)) and the lining (i = \( N^{*} +1, N^{*}+2, \ldots, N \)); \( \gamma \) – averaged value of the specific weight of rocks in the array (MN/m^3); \( H \) - depth of tunnel (m); \( l_{0} \) - is the lag of construction of the lining from the bottom of the tunnel (m).

2. Is determined by the value of the corrective multiplier: \( \alpha^{*} = \frac{1}{1 - \frac{3\lambda^{2}}{k_{0}} \gamma} \).

3. If you change the index \( i = 1, \ldots, N \) are auxiliary quantities: \( G_{i} = \frac{E_{i}}{2(1+\mu_{i})} \).

4. Putting \( c_{1} = 0 \) and modifying the index \( i \) in the range \( i = 1, \ldots, N-1 \) are calculated values

\[ c_{i+1} = \frac{R_{i+1}}{R_{i}}; \quad \xi_{i} = \frac{G_{i}^*}{G_{i+1}^*} \frac{1 - c_{i+1}^{2}}{1 - c_{i}^{2}} , \] and also given the designation \( \lambda_{n,m} = \begin{cases} 1, & \text{if } n = m, \\ 0, & \text{if } n \neq m \end{cases} \) sets the value of the parameter: \( q_{i} = \lambda_{i, N^{*}}^{*} \gamma H^{*} \).

5. If you change the index \( i = 1, \ldots, N \) are the values...


7. Ning Duyen Phong, 2016. Elastic-plastic model of the massif, which takes into account the change in the strength of the rock in the formation of workings for calculating the fastenings of the subway under difficult mining and geological conditions. Mountain information and analytical bulletin. – № 6. – P. 241-250.


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